Fracture of radially edge-cracked discs

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The brittle fracture of thin radially edge-cracked discs has been studied in three loading situations: edge opening, pin loading and diametral compression. Theoretical equations for these configurations are given and compared critically with experimental tests on polymethylmethacrylate samples. The edge-opening geometry was the best test overall, though all three systems were advantageous when compared with other common toughness tests because of their exact theory, simple sample preparation, facile machining, easy precracking, straightforward loading and low propagation forces.

1. Introduction
The measurement of fracture toughness, that is the resistance of a material to crack propagation, has been an important activity since Griffith [1] first derived the criterion for fracture of a centrally cracked infinite elastic sheet in tension, and used the solution to estimate the toughness of glass. Griffith's method was approximate because his experimental glass specimens were not large enough to be considered infinite [2]. Since that time, many different fracture tests have become available [3–5], including the tension of radially cracked discs [6], and numerous theoretical studies have been performed to interpret such tests [7–9].

Fracture tests may be divided into three broad types. The first category is mathematically simple, like the Griffith test or the adhesive fracture test for two contacting spheres [10]. However, such tests are often not practical because they require infinite samples or centrally placed starter cracks. A second type is the practical test such as the double cantilever, the double torsion or compression splitting geometry [11]. These test have been analysed intuitively, but require calibration to give an accurate interpretation of measurements. Finally, there are the much-used engineering tests such as bending or compact tension which are simple to perform but which are too complex for exact or simplistic analysis and which must be analysed numerically.

The problem is that the most convenient practical tests of fracture toughness are the worst to interpret theoretically. Numerical approximations may be adequate in most instances, but have been shown to give substantial errors in certain cases where small terms have been neglected [12]. The purpose of this note is to investigate this paradox of fracture testing by investigating the only finite geometry which has yet given a closed-form solution for toughness: the radially-edge-cracked disc [13, 14].

2. Criteria for fracture testing
Table I ranks a number of fracture toughness tests in terms of six practical criteria for success. These criteria involve specimen preparation, the performance of the crack propagation, and the interpretation of the test results in terms of toughness values. Each criterion scores a 1 for good or a zero for bad, and the six scores are added to assess the total ranking.

The first two columns relate to sample preparation in terms of easy machining and precracking. Extensive machining is expensive and is best avoided, as are central precracks which are difficult to insert. The next three columns deal with the cracking experiment itself, in relation to the ease of gripping the sample,

TABLE I Comparison of fracture test methods

Method	Score						
	Machining	Precracking	Grips	Load	Stability	Theory	Total
Griffith	1	0	1	0	0	1	3
Double cantilever	0	1	0	1	1	0	3
Compact tension	0	1	0	1	1	0	3
Bending	1	1	1	1	0	0	4
Compression	1	1	1	0	1	0	4
Double torsion	1	1	1	1	1	0	5
Indentation	1	1	1	1	1	0	5
Pin-loaded disc	0	1	1	1	1	1	5
Disc; compression	1	1	1	0	1	1	5
Disc; edge-opened	1	1	1	1	1	1	6

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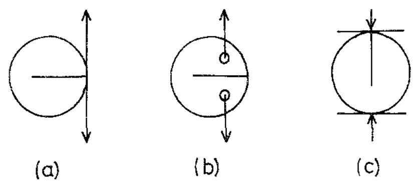

Figure 1 Three tests for fracture toughness of a radially-edge-cracked disc: (a) edge opening, (b) pin-loaded and (c) diametral compression.

the load required to drive the crack, and the stability of the crack either in speed or direction. Configurations which need complex grips, high cracking forces or rapid measurement of a fast-moving crack are undesirable. The next to last column of Table I assesses the ease of interpretation of the results in terms of cracking theory.

Consider the Griffith test according to these criteria. It uses a simple sheet and so there is little machining of samples. However, precracking is a nuisance because of the central crack and therefore scores zero. Gripping is easy with rigid clamps but large propagation forces are necessary and the crack goes with a bang because of poor stability, thus earning two more zeros. This test is somewhat redeemed by its good theory but only for samples which are large compared with the crack length. Overall, the Griffith test scores 3 on these criteria and is not ideal for toughness evaluation.

Looking down Table I, it is evident that standard tests like the compact tension and the bending tests are also not ideal. Double torsion and indentation methods are better but suffer from poor theory. The best geometries according to Table I are the edge-opened radially cracked disc and the compression-loaded disc. Therefore, these geometries were tested experimentally (Fig. 1).

3. Experimental procedure and results

Discs of diameter 50 mm and 3 mm thick were cut from standard polymethylmethacrylate sheet (ICI Perspex) using a diamond-coated tube which was rotated in a drill and pressed on to the sheet until a disc was cut out. To initiate the radial edge crack, a razor was pressed radially on to the edge of the disc until a crack could be seen penetrating the polymer.

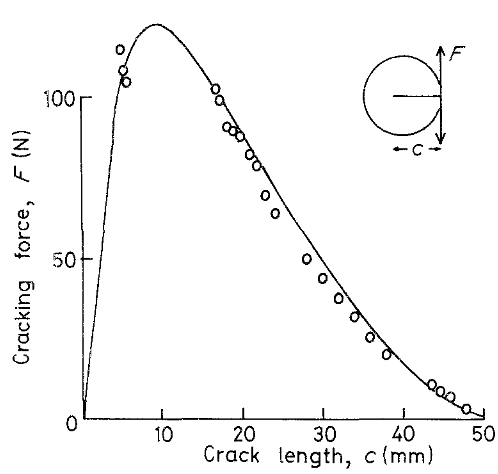

Figure 2 (O) Cracking results for edge opening of 3 mm thick disc, 50 mm diameter, containing a radial edge crack, compared with (——) theory of Equation 1 with $K_{Ic} = 1.12\,\text{MPa}\,\text{m}^{1/2}$.

This crack was driven for about 10 mm, when it tended to deviate from the radial path because of the wedging action of the blade. Longer precracks were introduced by propagating the short crack in double-torsion loading.

3.1. Edge opening

For the edge-opening test (Fig. 2) a slot was cut into the disc at the mouth of the precrack to accommodate steel tension arms which were pulled by the Instron 1122 machine. On applying a force of around 100 N to these arms, the precrack was observed to grow and the applied force was controlled by hand before catastrophic fracture occurred. Cracks shorter than 15 mm long were difficult to stop by this method, but cracks longer than this were readily arrested and restarted to obtain a series of propagation results on the one sample.

The force required to propagate the crack at a speed of $100\,\mu\text{m}\,\text{sec}^{-1}$ was measured at several points along the crack path diameter by referring to a millimetre scale drawn on the surface of the sample. These results were plotted in Fig. 2 for comparison with the exact solution given by Gregory [14],

$$\frac{F}{b} = K_{Ic}\left(\frac{c}{2d}\right)^{1/2}\left[\frac{c}{0.3557(d-c)^{3/2}} + \frac{2}{0.9665(d-c)^{1/2}}\right]^{-1} \quad (1)$$

where F is the force, c the crack length, K_{Ic} the fracture toughness of the disc, b its thickness and d its diameter. The results followed the theoretical equation reasonably well when the toughness was taken to be $K_{Ic} = 1.12\,\text{MPa}\,\text{m}^{1/2}$, the known value for polymethylmethacrylate under these conditions.

It was concluded that this test was convenient in all aspects. It required simple specimens with little machining and easy precracking; loading with simple grips at low loads gave a controllable crack, and the toughness was readily calculated from the exact theory.

3.2. Pin loading

Several samples were drilled to take 3 mm diameter pins which were then loaded in the Instron machine. The precrack was driven between the pins for a distance of 30 mm before propagation became easy to control. Even then, the force applied to the pins was higher than that expected from the equation [14]

$$\frac{F}{b} = K_{Ic}\left(\frac{dc}{2}\right)^{1/2}\left[\frac{c(d-a)}{0.3557(d-c)^{3/2}} + \frac{d-2a}{0.9665(d-c)^{1/2}}\right]^{-1} \quad (2)$$

where a was 14 mm, the distance of the loading pin centres from the tangent at the crack mouth. For cracks longer than 35 mm, the driving force fell considerably and gave a close fit to the theoretical line (Fig. 3). Evidently, this pin-loaded configuration is only suitable for testing discs with cracks extending through more than 70% of the diameter. The other

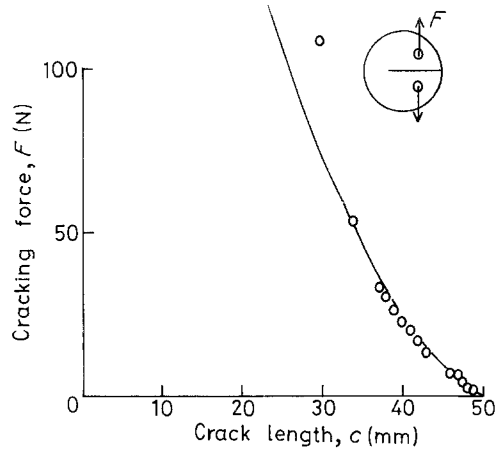

Figure 3 (○) Results for cracking of pin-loaded disc compared with (——) Equation 2 with $K_{Ic} = 1.12\,\mathrm{MPa\,m^{1/2}}$.

problems with this geometry are the extra machining needed to locate the pins, the higher loads needed to propagate the crack and the lack of exactness in the theory for this case. Equation 2 has been shown to be approximate, but gives an error of only 1% for long cracks [14]. Thus, the pin-loaded disc seems less suitable for fracture test purposes than the edge-opening mode, although it has been suggested as an ASTM standard [15].

3.3. Compression

Perhaps the most convenient experimental loading arrangement is compression (Fig. 4) in which a precracked disc is loaded between platens, to drive the crack diametrically across the disc. In the absence of a crack, the compressive force F produces a tensile stress $2F/\pi bd$ which is uniform across the plane of loading. This is the basis for the diametrical or Brazilian test originally devised for measuring the tensile strength of concrete [11, 16].

When a crack is present (Fig. 4), it may be shown theoretically that this diametral loading is equivalent to a uniform pressure $2F/\pi bd$ opening the crack faces, together with two crack closing forces $F/\pi b$ applied at the mouth of the crack. These crack closing forces would prevent crack propagation were it not for the interference of the crack faces which prevents closure. Such interference makes it difficult to calculate the precise lateral force at the crack mouth. However, if it is assumed that the lateral forces cancel, then cracking is driven only by the uniform pressure term and is described by

$$\frac{F}{b} = K_{Ic} d \left(\frac{\pi}{2c}\right)^{1/2} \frac{[1 - (c/d)]^{3/2}}{1.586} \tag{3}$$

This equation was tested experimentally by compressing precracked polymethylmethacrylate discs of diameter 50 mm and 3 mm thick in the Instron testing machine. The results are shown in Fig. 4 for comparison with the theory.

For short precracks, around 10 mm in length, it was impossible to propagate the crack even at a load of 1400 N because the material flowed plastically around the loading points. Longer precracks, about 16 mm in length, did allow propagation but the loads were an order of magnitude higher than in the edge-opening test, as expected from the theory.

Two discrepancies were observed in the results. In the first place, the propagation forces were significantly higher than anticipated from Equation 3. This may have stemmed from the large plastic regions which were seen near the loading points. These plastic zones were some 3 mm in diameter and were not considered in the theory. The second discrepancy became apparent when the length of the crack reached 25 to 30 mm. The cracking force levelled off with crack extension and then the crack suddenly became unstable and accelerated rapidly. It was suspected that this behaviour was due to the constraining action of the rigid loading plate which was preventing crack opening. The theory presumes that there is no lateral constraint at the crack mouth.

In order to alleviate these two difficulties, the precrack was lengthened to around 20 mm, thus preventing excessive plastic flow at the loading point, and the rigid loading plate was split into two steel fingers as shown in Fig. 5, permitting lateral separation at the crack mouth. The steel fingers were each 1.5 mm thick, 6 mm wide and 30 mm in length. With this arrangement the crack propagation was more reproducible and the instability disappeared, though the results indicated a substantial lateral constraint at large crack lengths.

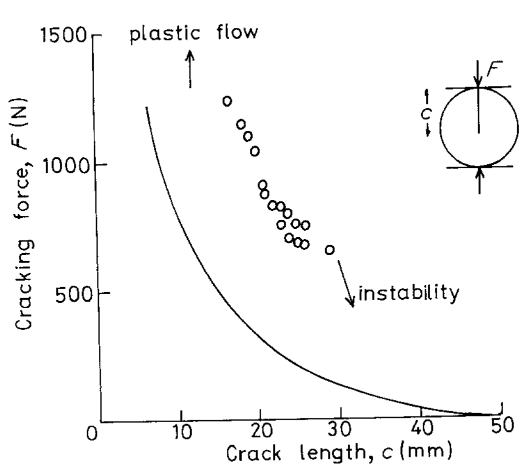

Figure 4 Discrepancy between (○) cracking results for compressed disc and (——) Equation 3, showing the plastic domain at high loads, the high propagation force at intermediate crack lengths, and the instability for long cracks.

4. Conclusions

The only finite edge-crack geometry with a closed form solution for K_{Ic} is the radially edge-cracked

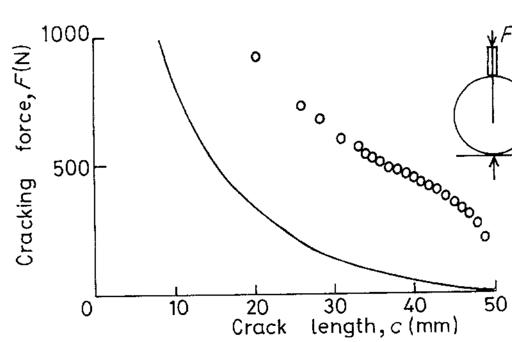

Figure 5 Results showing the improved performance of the compression cracking test with the split finger loading device. (○) Measured values; (——) Equation 3 ($K_{Ic} = 1.12\,\mathrm{MPa\,m^{1/2}}$), which does not describe the results at large crack lengths.

disc. This geometry can form the basis of a convenient practical toughness test in three loading modes: edge opening, pin loading and diametral compression.

Experiment has shown that the edge-opening configuration gave the best overall performance for the cracking of polymethylmethacrylate discs. Precracks longer than 30% of the disc diameter proved to be readily controllable, and the propagation loads fitted the theory rather well. By contrast, the pin-loaded disc was only useful for cracks longer than 70% of the diameter. In compression, there were problems of plastic deformation at the loading points, plus instability arising from lateral constraint. The theory requires substantial modification for this case.

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